John Venn (1834 – 1923) was born in Hull England and came from a family of distinction. Not only was his father Henry a member of the clergy, but also his grandfather John (for whom he was named). Both men were prominent in the evangelical Christian movement of the time, which was influential in terms of both societal reform and missionary work. Given this family background, Venn had a strict upbringing and there was little doubt he would follow the family tradition into the priesthood. While Venn did indeed enter the priesthood shortly after graduating from Cambridge University in 1857, serving as curate briefly in two parishes, he held a wide range of interests. Venn, upon graduating earned the position of 6th "Wrangler" (6th among the First Class winners) on the famous Mathematical Tripos examination and in 1883 he was elected a fellow of the Royal Society of London. [1] Though he maintained his vows for over 20 years before stepping away from the clergy under the terms of the Clerical Disabilities Act of 1870, he was already back at Cambridge by 1862 where his title was Lecturer in Moral Sciences. There he studied and taught Logic and Probability Theory and also developed an interest in Philosophy and Metaphysics. Interestingly, he never viewed his exploration of these interests to be at odds with his religion. [3]

His son, John Archibald Venn explained the position in writing his father’s obituary:

“It had long ceased to be regarded as an anomaly for a clergyman to
preach the then circumscribed evangelical creed and at the same time, without
the slightest insincerity, to devote himself actively to philosophical studies; yet ...
finding himself still less in sympathy with the orthodox clerical outlook, Venn availed himself of the Clerical Disabilities Act. Of a naturally speculative frame of mind, he was want to say later that, owing to subsequent change in accepted opinion regarding the Thirty-nine Articles, he could consistently have retained his orders; he remained, indeed, throughout his life a man of sincere religious conviction.” [4, pp. 869-870]

While Probability obviously involves numbers, and determining probability is generally regarded as a mathematical exercise, Venn’s approach in his 1888 3rd edition of The Logic of Chance is straight-forward and purposely non-mathematical. This approach is taken not only in the interest of making it accessible to those without an extensive mathematical background, but also due to his belief that treating the subject as a “portion of mathematics” sells it short. Instead, he argues that the study of Probability is more properly a branch of the general science of evidence that happens to make use of mathematics. [5, p. vii]

Venn does however acknowledge from the outset that the study of processes and laws (such as Probability) is indeed more challenging by its nature than the study of sciences dealing with physical objects, regardless of approach. For instance, if the reader is generally familiar with the objects being studied, referencing them can at least give a tolerable understanding of the direction and nature of the study. Conversely, the study of a process or law with which the reader is not already familiar presents little opportunity to provide prior information. The reader must be “taken in hand” and directed to an initial, basic understanding of the subject matter before getting into the details. [5, pp. 1-2]

While Venn’s approach is indeed “approachable” for those uninitiated in Probability, Frequency Theory does rely more on calculation and computation compared to Classical Theory,
which is driven largely by the Principle of Indifference (events are equal and indistinguishable; probability of each is 1/n). Frequency Theory on the other hand defines the probability of an event occurring to be its actual observed relative frequency based on a large number of trials. A standard example of the type of problem to which the Principle of Indifference can be effectively applied is a dice problem. Assuming symmetrical six-sided dice, there is no reason to believe that any number (1-6) would come up more often than any other number. On this example: [2 p. 643]

“In certain types of games, the early practitioners were able to work out efficient ways of counting successes and failures and thus to determine the expectation or probability a priori [before hand]. In most realistic situations, however, it was much more difficult to quantify risk, that is, to determine the degree of belief that a “reasonable man” would have. How could one determine a “reasonable” price to pay for insurance?” ...

Jakob Bernoulli, in his study of the subject over some 20 years, wanted to be able to quantify risk in situations where it was impossible to enumerate all possibilities. To do this, he proposed to ascertain probabilities a posteriori [afterward] by looking at the results observed in many similar instances, that is, by considering some statistics.”

As detailed below, this is the exact direction Venn headed with his work. That is to say, an approach that was comfortably ignorant of the details, but secure in the averages...particularly over a large number of trials.

This type of natural symmetry also supports the leaps of Inductive Inference associated with Classical Logic. Venn’s position, as touched on below, considers Induction simply as adjunct to Probability.
Venn begins by addressing classes and clarifying the classes with which the science of Probability is concerned. On the one hand, there are situations where a general proposition can be equally true of both the whole and the component parts – if all cows ruminate (the process of chewing food for a second time, also known as “chewing the cud”), then any particular cow or group of cows ruminate. On the other hand, if we know that only some cows ruminate, it is not possible to infer the general from the particular in the same way. Though it would be reasonable to assume that an individual cow might ruminate, it cannot be logically inferred. However, one could more assuredly infer that in a given herd of cows, some are ruminant – basically, there is uncertainty about individuals of the class, but certainty about the class as a whole increases with the size of the class. This latter larger class of things being the focus of the science of Probability. [5, pp. 2-3]

This concept carries over to his explanation of a particular kind of series, a series combining individual irregularity with aggregate regularity. As well, he speaks to the key distinction between absolute irregularity and relative irregularity. The well-known example of tossing a coin serves to illustrate both. For instance, when observing a small number of tosses, the series often appears to be random and chaotic (3 tails, then 1 head, then 2 more tails, etc.) This is individual irregularity. But, as the number of tosses increases, order begins to emerge and the number of heads and tails begins to equalize. This is aggregate regularity. And, we know that over a large number of tosses the relative irregularity is negligible – i.e. the proportion of each will be very near 50%. At the same time, as the number of tosses increases so does the absolute irregularity - i.e. the *numerical* difference between total heads vs. tails. Finally, series in this sense does not denote consecutiveness, but rather a number of observations (order of observed
occurrences is immaterial), with order emerging out of disorder as the number of observations increases [5, pp. 4-13]

In examining the “logical superstructure” underlying the physical foundations Venn has established, he begins with consideration of the vital role of belief and how it supports drawing inferences. He describes this consideration as a shift in focus from the objective measurement of things to the subjective manner in which we contemplate them. One immediate impact of our subjective consideration is generalization – a process of substitution and idealization by which series are described in terms of their aggregate regularity at the expense of recognition and visibility of individual irregularity. An example of the type of immediate inference that a high level of belief allows: “All men are mortal, therefore any particular man or men are mortal.” [5, pp. 119-121]

Next, with regard to the ultimate impact of belief, Venn points to the proposition that while belief is variable (sometimes highly), it cannot be accurately measured. This position is in contrast to that of a number of his contemporaries, who seem to suggest that belief is measured by Probability as opposed to being a determinant in how we go about forming inductive inferences. He specifically cites a Prof. Donkin (Phil. Mag. May, 1851):

“It will, I suppose, be generally admitted, and has often been more or less explicitly stated, that the subject-matter of calculation in the mathematical theory of Probabilities is quantity of belief.”

Regarding a specific measurement of the amount of belief, Venn notes the impact of strong emotions (passion), as well as the effect of extreme complexity and variety in the evidence upon which beliefs depend. As a basic example of this he suggests the phenomenon of lotteries and the
fact that an individual’s related beliefs and actions (e.g. buying a ticket) are not consistent with
the underlying Probability assigned by theory. [5, pp. 123-128]

From the type of inference discussed in relation to the discussion on belief – inferences
regarding particular propositions based on the general propositions that include them, Venn
moves on to the rules governing inference more broadly. That is, an examination of the cases in
which one general proposition can be inferred from another. He begins by drawing a distinction
between what he calls the formal, or fundamental, rules governing inference in Probability and
those rules that are more experimental, or experiential, in nature. He deems the latter the subject
of Induction. The fundamental rules, those based on the mere application of arithmetic, fit into
two categories.

The first category dealing with exclusive or incompatible events, governs inferences
drawn based on addition or subtraction. For instance, if \( \frac{364}{100} \) infants live to over sixty and
another \( \frac{354}{100} = \frac{3540}{10000} \) die before they are ten, then out of a group of say 10,000, it can be inferred
that approximately \( 2820 = 10000 - (3640 + 3540) \) will live between ten and sixty years. The
general algebraic rule for the probability of one or the other of two incompatible or mutually
exclusive events happening is \( \frac{1}{m} + \frac{1}{n} = \frac{m+n}{mn} \) where \( \frac{1}{m} \) and \( \frac{1}{n} \) are the probabilities of the respective
events. This can also be applied when only one said event will happen by subtraction instead of
addition resulting in the expression \( \frac{1}{m} - \frac{1}{n} = \frac{n-m}{nm} \). Also when talking about the probability of an
event not happening you would subtract the chance of the event happening from one, \( 1 - \frac{1}{m} = \frac{m-1}{m} \). [5, pp. 167-171]

The second category dealing with dependent events, governs inferences drawn based on
multiplication or division. For instance, if \( \frac{25}{1000} \) London inhabitants will die in the course of the
year and \( \frac{1}{5} \) of those deaths will be due to fever, then it can be inferred that \( \frac{1}{2000} = \frac{1}{5} \cdot \frac{2}{1000} \) of the inhabitants will die of fever in the course of the year. The general equation here would be the chance of one event happening and a second event happening as well is \( \frac{1}{mn} \). It then follows that:

\[
This \text{ rule, expressed in its most general form, in the language of }\
\text{Probability, would be as follows: - If the chances of a thing being } p \text{ and } q \text{ are}\\
\text{respectively } \frac{1}{m} \text{ and } \frac{1}{n}, \text{ then the chance of its being both } p \text{ and } q \text{ is } \frac{1}{mn} = \frac{1}{m} \cdot \frac{1}{n}, p\\
\text{and not } q \text{ is } \frac{n-1}{mn} = \frac{1}{m} \cdot \left(1 - \frac{1}{n}\right), q \text{ and not } p \text{ is } \frac{m-1}{mn} = \frac{1}{n} \left(1 - \frac{1}{m}\right), not \ p \text{ and not}\\
q \text{ is } \frac{(m-1)(n-1)}{mn} = \left(1 - \frac{1}{m}\right) \left(1 - \frac{1}{n}\right), \text{ where } p \text{ and } q \text{ are independent.}\
\]

Venn’s primary focus, as described above, is on the type of inferences that are calculated, as opposed to those based solely on Induction. However, he does not as a result simply ignore Induction – he recognizes it for what it is and clearly defines where it does (and doesn’t) fit into his approach to Probability. Without getting too deeply into something that is a subject unto itself, a couple of statements really sum up where he draws the line: [5, p. 208]

“\text{It would be more correct to say, as stated above, that Induction is quite distinct from Probability, yet co-operates in almost all it inferences. By Induction we determine, for example, whether, and how far, we can safely generalize the proposition that four men in ten live to be fifty-six; supposing such a proposition to be safely generalized, we hand it over to Probability to say what sort of inferences can be deduced from it.}”

So, Induction not a part of the science of Probability, but complementary and a useful “place to start” in many instances.

When it comes to reconciling his take on the science of Probability with the seemingly incompatible concepts of causation and design, Venn takes a position that seems to indicate at
least some level of surprise that they are viewed as incompatible. He starts off with what he refers to as a very old Theological objection – something he should have some intimate perspective on! While noting the religious perspective on attributing matters to chance is actually centuries older than the Theory of Probability itself, he notes: [5, p. 235]

“If we spelt the word [causation] with a capital C, and maintained that it was representative of some distinct creative or administrative agency, we should presumably be guilty of some form of Manicheism (a religion based on dualism).”

Venn states matter-of-factly that the science of Probability is actually mute on causality, “making no assumption whatever” about the way events come about. Instead, the concern of Probability is establishing a set of rules, with those rules applicable to classes of cases where making definitive inferences about the individuals comprising the classes is not possible. So, the chance (odds) of an event occurring is simply a statement regarding the average frequency with which it would be expected to occur, not a statement that the event is brought about by chance (meaning lacking causation). [5, pp. 235-236]

In his analysis of the insurance example from the chapter titled “Insurance and Gambling” Venn convincingly shows that while it (insurance) is widely believed to be good and proper, compared to gambling which is seen as a moral shortcoming by many, they are very much the same as far as Probability is concerned. His initial supposition is, that at their core, they are clearly applications of the very same Probability concepts – individual irregularity and average regularity. Beyond the moral implications associated with gambling, an individual’s attraction to one or the other is simply a matter of that individual’s personal perception of, and level of comfort with risk. He goes on to point out how the diminishment of the general level of
risk in society, driven by various modern developments, amplifies not only the differences in perception, but also the consequences of the risk that remains. [5, pp. 370-371]

Using life insurance as a case in point for insurance generally, Venn frames it in simple terms. Those desiring relief from the uncertainty regarding length of life and time of death basically agree to “make up a common purse” with others sharing the same concern. Those individuals who end up having a longer than average life span effectively contribute to the support of the families of those who die earlier. The principle of average regularity dictates that the greater the number of individuals contributing to the purse, the greater the certainty of achieving the desired relief. He points out that the particulars, such as fixed annual premiums, are mere accidents of convenience and arrangement. Underlying the propositions is the knowledge and understanding that for one to gain, another must lose an equal amount (or many must lose a little). Thus, for the individuals involved, while it isn’t “fair” in terms of the economic outcome, it’s worth it. [5, pp. 372-373]

Venn’s examples dealing with both insurance and gambling are fairly classic targets for the application of Probability Theory, particularly Frequency Theory. There are other matters however, where although there is a similar need to make a determination regarding the chances of occurrence, the event/subject does not lend itself to the large sample sizes seen with insurance or gambling. His chapter dealing with the “Credibility of Extraordinary Stories” addresses an example of this other, more ad hoc, class of events. He set it up as follows: [5, p. 406]

“It will be remembered that in a previous chapter (the twelfth) we devoted some examination to an assertion by Butler, which seemed to be to some extent countenanced by Mill, that a great improbability before the proof might become but a very small improbability after the proof. In opposition to this it was pointed out that the different
estimates which we undoubtedly formed of the credibility of the examples adduced, had nothing to do with the fact of the event being past or future, but arose from a very different cause...”

This cause is identified as the way in which the conception of an event comes about. The first way being a mere guess of our own, which based on what we know from related statistics, would be right in certain proportion of cases. The second way being the assertion of a witness, which is indeed very different in that it is not an appeal to statistics, but rather to the veracity of the witness. [5, p. 406]

Venn continues by posing the question as to whether or not this transfer of focus then means that the probability or improbability of an assertion depends solely upon the veracity of the witness. And, if so, by extension, would any story told by a veracity of the person (truthful 9 of 10 times for instance) then be believed? What he suggests is, no, not without a bit of fine tuning. As with the case of life insurance, even though statistics and averages are the proper approach, going beyond the general average (incorporating occupational and lifestyle factors), when possible, will yield a more accurate result. So, in the case of a witness providing testimony on something extraordinary: [5, pp. 407-409]

“Cannot we, in almost any give case, specialize it by attending to the various characteristic circumstances in the nature of the statement which he makes; just as we specialize his prospects of mortality by attending to circumstances in his constitution or mode of life? Undoubtedly we may do this; and in any of the practical contingencies of life, supposing that we were at all guided by considerations of this nature, we should act very foolishly if we did not adopt some such plan.”
While one method of making the needed correction to a general average, one that Venn describes as a “conjectural correction” based on “practical sagacity” – essentially based simply on past experience and observation, might be more in keeping with the approach of Classical Theory, he expands on a second method focused instead on a more rigorous analysis of the nature and number of sources of error. This approach considers factors such as whether the testimony given requires a yes or no answer to a question (only one way to be wrong), as opposed to an open-ended response (many, possibly endless ways to be wrong). [5, pp. 409-411]

So, taking again the witness known to be truthful 9 of 10 times, Venn presents the following of an example of how to further determine the likely veracity of the witness’s testimony in a scenario with one way to be wrong. Specifically, drawing balls from a bag, of 10,000 balls in which 10 are white and 9990 are black:

“In the 10,000 drawings the white ball would come out 10 times, and therefore he rightly asserted [guess white correctly] 9 times out of ten [based on his above mentioned truthfulness], whilst on the one of these occasions on which he goes wrong he has nothing to say but “black”. So with the 9990 [10,000 − 10] occasions on which black is drawn; he is right and says black on \[9990 \cdot \frac{9}{10}\] of them, and is wrong and therefore says white on \[999 \cdot \frac{1}{10}\] of them. On the whole, therefore, we conclude that out of every 1008 [999 + 9] times on which he says that white is drawn he is wrong 999 times and right only 9 times. That is, his special veracity, as we may term it, for cases of this description, has been reduced from \[\frac{9}{10}\] to \[\frac{9}{1008}\].”

The generalized form of the above example follows that if he asserts that the event happened, the probability that it does happen is \[\frac{px}{px+(1-p)(1-x)}\], where \(p\) is the probability of an
event and $x$ is the veracity of the witness. Here both $0 \leq p \leq 1$ and $0 \leq x \leq 1$ where $x = 0$ means the witness is never truthful and $x = 1$ means that the witness is always truthful. $(1 - p)$ is the probability that the event does not occur and $(1 - x)$ is the probability that the witness is not truthful. In the example quoted above this appears as $\frac{9}{1008} = \frac{9}{10,000} \times \frac{10}{9}$. Here $p = \frac{10}{10,000}$, $(1 - p) = \frac{9990}{10,000}$, $x = \frac{9}{10}$ and $(1 - x) = \frac{1}{10}$. However, if the witness asserts that it did not happen the probability is $\frac{p(1-x)}{p(1-x)+(1-p)x}$. Where once again the same rules for $(1 - p)$ and $(1 - x)$ from above apply. [5, pp. 411-414]

Given Venn’s family history and background, his work on Probability (particularly the Frequency Theory approach) is notable. At a time when the study, acceptance, and championing of such topics was seen as incompatible with Theology, he appears to have moved easily between the two. Regarding causation (including the possibility of intelligent design) he states: [5, p. 239]

“On the theory adopted in this work we simply postulate ignorance of the details, but it is not regarded as of any importance on what sort of grounds this ignorance is based. It may be that knowledge is out of the question from the nature of the case, the causative link, so to say, being missing. It may be that such links are known to exist, but that either we cannot ascertain them, or should find it troublesome to do so. It is the fact of this ignorance that makes us appeal to the theory of Probability, the grounds of it are of no importance.”

As noted earlier, John Venn was a man who enjoyed a wide range of interests. This brief examination of his work in the area of Probability Theory shows that he certainly had the intellect to explore these interests in considerable detail. As well, he seemed to have no real
difficulty reconciling apparent contradictions existing among those interests. The passage above suggests that ignorance of the details, including causation, drives us to the study of Probability to gain understanding. But, as he made clear in his chapter “Chance, Causation, and Design” (and as touched on previously above), this appeal to the study of Probability concerns chance as an indicator of the odds of event occurrence, totally separate from the concept of chance as a causal factor.

So, in conclusion, Venn is rightly acknowledged as a key contributor to the development of the Frequency Theory of Probability. Based on a more rigorous, mathematical approach, the theory certainly provides greater accuracy of prediction compared to Classical Theory methods, particularly when a large number of events are available for analysis. However, that’s the “what” – Venn’s work is arguably more influential, to this day, due to the “how” of his approach.

While Venn focused on sound computational methods, he was careful not to discount influences such as belief, intuition, and human emotion. Each of these factors he considered and took into account, largely without disparagement or dismissal. And, he did this in an era when there was a fairly distinct division between scientific thinking and religion which was a determinant of who Venn ultimately was. Perhaps more importantly, recognition of how methods such as Inductive Inference, borrow from Classical Theory and Formal Logic, could co-exist with, and be complementary of, Frequency Theory was key. Finally, as is the case with the famous Venn Diagrams, in the Logic of Chance he does it all in a way that is widely relatable.
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